NONLINEAR BIOMEDICAL PHYSICS

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Econobiophysics - game of choosing. Model of selection or election process with diverse accessible information

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Abstract

We propose several models applicable to both *selection* and *election processes* when each selecting or electing subject has access to different information about the objects to choose from. We wrote special software to simulate these processes. We consider both the cases when the environment is neutral (natural process) as well as when the environment is involved (controlled process).

Keywords: econophysics, preferences, impact, selection process, mating, monogamic, polygamic, election campaign, spin, influence of environment

Introduction

We consider a system $\mathbf{U}$ consisting of two classes of elements - a class $\mathbf{S}$ of *subjects* that are *active*, that is they can make decisions to choose (to select or to elect) elements they somehow prefer from a class $\mathbf{O}$ of *objects*. Objects are *passive* i.e. they can only compete to be chosen (selected or elected). Subjects and objects are embedded in a much greater space called *environment*, $\mathbf{E}$ (cf. [1]).

Let us assume that the class $\mathbf{S}$ consists of N subjects A, B, C, ... and the class of objects $\mathbf{O}$ consists of M subclasses α, β, γ, ... with i_m objects belonging to the mth subclass (m = 1, ... , j_m) - j_1 objects a belonging to subclass α, j_2 objects b belonging to subclass β, etc. In the simplest case each subclass consists of only one object.

Subjects' preferences are given in the form of *preference matrix $P = [p_{mn}]$* where the element p_{mn} denotes preference of the n-th subject towards m-th subclass of objects.

We call the process in the considered system $\mathbf{U}$ the *Game of Choosing, GoC*. GoC is characterized by discrete time, t, that counts number of rounds of the GoC from the initial moment. Each round means *exposition* of the objects to the subjects. The periods between the subsequent rounds may differ from one another. Exposition may be direct (one may call it *sensorial*) presentation of

the object to the subject or it may be just a transmission of information about the object to the subjects through the environment, for example presentation of a TV spot. For a subject to win GoC means to choose the most preferred object before other subjects would do this or to choose more objects than other subjects during the same period of time. For an object to win GoC means to be chosen before other objects would be.

Subjects and objects exchange information through the environment. The environment may be *neutral*, i.e. it does not influence transmitted information; in such a case we call the process *natural GoC* as for example when *choosing a mating partner*. The environment may be *active*, i.e. it can somehow transform information it transmits; in such a case we call it *controlled GoC* as for example when *choosing a product* based on information added by Google every time one searches the web or uses Gmail or when *electing an MP* influenced by public relation (PR) propaganda.

For simplicity we will assume that each subclass consists of only one object, so there are M objects and N active subjects. Subjects' preferences are equal to probability of choosing given object multiplied by 100. If p_{mn} = 50 it means '50-50' situation, that is equal probability of choosing and of rejecting object m by subject n. If $p_{mn} = p_t = 80$ it means that probability of choosing the object m by subject n is four-times greater than probability of rejecting it.

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In the beginning we initialize preference matrix P with values close to 50, between 48.5 and 51.5, i.e. all subjects have very similar preferences to all objects but some preferences are slightly greater than others (Figure 1). We assume that the influence of the objects on the subjects is expressed in the form of the *impact matrix I = [i_{mn}]* where the element i_{mn} denotes influence of one exposition of the m-th object on the preference of the n-th subject towards this object (Figure 2). And the evolution of the system in time is given by

$$p_{mn}(t+1) = p_{mn}(t) * i_{mn}(t) \qquad (1)$$

by multiplying element by element (not multiplication of the matrices!), where t denotes the number of rounds of GoC. If $i_{mn}(t) = 1$ it means that the exposition does not change the preference of the n-th subject towards the m-th object, what is equivalent to the assertion that the m-th object was not at all exposed to the n-th subject in the round t. It makes possible to adopt a simplifying assumption that all objects are exposed to all subjects during each round of GoC.

If $i_{mn}(t) > 1$ it means that the exposition number t causes increase of the preference of the n-th subject towards the m-th object; if $i_{mn}(t) < 1$ the exposition causes decrease of the preference. So, the state of the system after t rounds of GoC depends on subjects' initial preferences as well as on the impact matrix and its evolution in time.

We consider several variants of GoC with different interactions between subjects, objects, and environment. Our models may be applied both in biomedical systems as well as in socio-economical systems. By analogy to *econophysics* we will call it *econobiophysics*.

In the matrices columns correspond to subjects and rows correspond to objects, and the 'compartment' at the crossing of column n with the row m that may be filled up with a value of the corresponding element will be called the *cell*. Preference matrix cell is of green shade if the preference is higher than 50 and otherwise it is of red shade. Impact matrix cell is of green shade if the impact is higher than 1.0 and otherwise it is of red shade. So, in the figures and in the moving pictures (cf. additional files) green shade signifies that the m-th object is being chosen by the n-th subject and red shade that it is being rejected. We generate starting p_{mn} values to be close to 50 - between 48.5 and 51.5 and i_{mn} values to be close to 1 - between 0.99 and 1.01. The game starts from a random point which is marked with more intensive colour then others. In the following example (Figure 1 and Figure 2) the game starts from the 9-th row and the 5-th column.

	1	2	3	4	5	6	7	8	9
1	50.4391	50.175	50.2127	50.7288	51.011	49.7618	50.394	49.0383	50.6666
2	51.0247	49.4005	50.5561	50.1977	49.9087	50.3878	49.0204	49.5446	50.8165
3	49.426	50.189	50.3034	49.1711	50.2248	49.9162	50.5546	50.3585	49.0404
4	49.3588	49.0637	50.0987	50.949	48.782	49.8066	50.7551	49.6311	50.1686
5	49.3981	51.136	49.0455	50.0144	50.4842	51.0717	48.8055	49.2606	50.1998
6	50.8104	48.9796	50.1446	50.6761	50.518	50.3468	49.7352	50.5235	49.8751
7	50.4189	48.6842	50.7784	50.9273	48.6807	50.9361	51.0439	50.5135	50.8347
8	50.2981	49.0693	51.323	50.0094	50.6701	49.4287	50.6192	49.9429	50.2422
9	50.541	50.7963	50.8575	49.5851	49.9748	50.4068	50.272	49.8161	50.0221
10	50.259	49.6924	51.0808	50.9786	49.4687	49.4919	49.1431	50.3826	50.7457
11	51.1694	49.6374	48.9438	51.4585	49.2509	51.0614	49.9047	49.8051	48.7506
12	50.2158	51.087	49.1437	50.9003	50.1016	49.7131	50.0338	49.6708	50.5135
13	51.2211	49.3306	48.7406	49.2358	50.2001	49.7344	50.4312	50.1676	50.1371
14	50.5853	49.0928	49.8271	50.0345	49.1799	51.0399	50.832	50.1556	51.2193
15	49.0466	50.0298	49.9925	49.6881	49.2245	50.437	50.6643	48.9035	50.2121
16	51.0627	50.0874	49.4646	50.4274	50.8767	49.9479	49.9854	51.0415	49.7354
17	49.2888	49.9522	49.7742	49.3685	49.9111	48.7085	49.4848	50.8711	50.4934
18	50.2065	50.1285	50.0139	49.6661	49.5715	50.5218	50.2348	51.2362	49.8142
19	50.0732	48.9626	49.9464	49.6118	48.9213	49.514	50.3237	49.8367	51.3149

Figure 1 Example of preference matrix.

	1	2	3	4	5	6	7	8	9
1	1.00247	0.998569	0.99903	1.00131	1.00811	0.9972	1.0073	0.99905	1.00077
2	1.00931	1.00668	0.993852	1.00358	1.0069	1.00083	0.996606	1.00434	1.00592
3	1.0018	1.00938	1.00624	0.990201	0.997426	0.999821	0.99479	1.00892	0.997821
4	1.00166	0.998436	1.00486	1.00044	0.992573	1.00665	0.99815	0.995719	1.00836
5	1.0041	1.00551	0.99781	1.00972	1.00446	1.00408	0.991414	0.993687	1.0043
6	0.999119	0.990023	0.99951	1.00018	0.995113	0.995189	1.00786	0.992031	0.990825
7	1.0087	0.992442	1.00919	1.00544	0.992252	1.00046	1.00768	0.991282	1.00127
8	1.00437	0.998453	1.00735	0.992955	1.00294	0.998588	1.00483	1.00431	0.990443
9	1.00379	1.00411	1.00501	1.00259	0.994148	1.00039	0.994663	1.00687	0.994245
10	0.996758	0.999443	1.00219	1.00345	0.995317	0.994285	0.991267	0.991643	1.00411
11	1.00995	0.999343	0.990571	1.00951	0.994497	1.00835	1.00061	1.00594	0.994801
12	1.00808	1.00514	0.996018	0.999162	1.0075	0.992664	0.997459	1.00831	0.99671
13	1.00654	0.994118	0.99261	0.999777	0.997576	1.00327	1.00385	1.00063	1.00476
14	0.998425	0.993936	1.00473	1.00754	0.999214	1.00899	1.00474	0.992149	1.00656
15	0.991323	1.00798	1.00206	0.991276	0.992613	1.0049	0.998559	0.991296	0.999256
16	1.0091	1.00821	0.992648	0.994233	1.00272	1.00624	1.00114	1.00562	0.996461
17	0.990374	1.00359	1.00965	0.990361	1.00182	0.9907	1.00768	0.997871	0.998106
18	1.00309	0.997903	1.00872	1.0051	0.991703	1.00898	1.0084	1.0059	0.992344
19	0.990476	0.993751	0.994362	1.00337	0.994247	1.00794	1.00176	0.994692	1.00684

Figure 2 Example of impact matrix.

To each model below the corresponding short moving picture showing the course of GoC is added in an additional files that for convenience we will call 'Picture Q.' where Q is the serial number of the Figure to which it corresponds (so there are no Picture 1. nor Picture 2. between the additional files). Each moving picture illustrates dynamics of the GoC, presented as changing of cells' colours with time. The end of each game is shown in the corresponding figure and when a number pops up in a cell it shows how many rounds of GoC have been necessary for the subject n to make decision to accept (green colour) or to reject (red colour) the object m. The colour of the cells and its intensity changes during the course of the game, with increasing intensities corresponding to increasing probabilities of acceptation or rejection - the greater is the intensity of colour the closer is the subject to final decision of choosing or rejecting the object. Intensity of colour changes instantly if the subject makes the decision. For convenience, only the 'winning/loosing' numbers are displayed at the end of the game showing how many rounds (expositions) each subject needed before making the decision; these numbers are also shown in the corresponding figure...

Results

I. Natural GoC - 'monogamic model'

In the simplest model we assume that the environment is neutral and the impact matrix I does not change with time; in such a case the results of selection process depend only on the number of times the objects reveal their impact on the subjects. At the moment when some preference p_{mn} exceeds the threshold p_t both the subject n and the object m stop to take part in the GoC while the game may continues until either all subjects would make their choice or until all available objects would be chosen. Natural selection 'monogamic model' is illustrated in Additional File 1. Picture 3. that results with Figure 3.

In this example subject 1. chose object number 11. after 47 rounds, while subject 8. needed 59 rounds to choose object 12. 10 out of 19 subjects lost the game i.e. they were not chosen at all.

II. Natural GoC - 'polygamic model'

The difference between the 'polygamic model' and the 'monogamic model' lies in the assumption that at the moment when some preference p_{mn} exceeds the threshold p_t only the object m stops to take part in the GoC while

Figure 3 GoC - monogamic model.

the subject *n* remains in the game and GoC continues. So, each subject may choose more than one object and at the end of the game all objects are chosen. Natural selection 'polygamic model' is illustrated in Additional File 2. Picture 4. that results with Figure 4. Object 10. is not being chosen for quite a long period to be finally rejected by subjects 7. and 8., but at the same moment it is chosen by subject 9. as the last free object of all, so the game is over - subjects 4., 5., and 7. have only one object each, while subject 1. has as many as four objects.

III. Natural GoC with interaction between subjects
In this model subjects do not act independently. The values of preference matrix for subject *n* change depending on the preferences of the nearest neighbours subjects (cf. [2]) and the evolution of the system in time is given by Eq. (2) instead of Eq. (1):

$$p(m,n) = \frac{\sum_{k=-a}^{a} \sum_{l=-b}^{b} i(m+k, n+l) * p(m+k, n+l)}{\sum_{k=-a}^{a} \sum_{l=-b}^{b} i(m+k, n+l)} \qquad (2)$$

where $a = (g\text{-}1)/2$ and $b = (h\text{-}1)/2$; $g*h$ is the size of mask defining the nearest neighbours. For convenience,

in the above formula we used indices in parenthesis instead of lower indices, so $p(m, n)$ means the element p_{mn} of the preference matrix, $i(m, n)$ - the element i_{mn} of the impact matrix, and the multiplication in the numerator means multiplying element by element. As in Eq. (1), the left-hand side of Eq. (2) corresponds to the round *t+1* of the GoC, while right-hand side to the preceding round *t*. In the following example the mask is 1 × 5. Toroidal 'boundary condition' are assumed for both preference and impact matrix so every subject and every object has two nearest neighbours.

Each subject tries to be as similar to its neighbours as possible so the preferences values $p_{mn}(t+1)$ increase relatively to $p_{mn}(t)$ for all objects belonging to the subclass j_m if preferences of the neighbouring subjects towards objects belonging to this subclass are greater than the preference $p_{mn}(t)$ of the considered *n*-th subject. Subjects make their choices depending not only on their own but also on their neighbours' preferences. In monogamic model (Figure 3.) subject 7. chose object number 18. In this case for model with one subclass (see Additional File 3. Picture 5., and Figure 5.) subject 7. chose objects 1. because of 'friendly neighbourhood'. Six out of nine subjects rejected object 6. For the model with the number of subclasses at least equal to the number of

Figure 4 GoC - polygamic model.

subjects every choice is rather choice of the whole group (see Additional File 4. Picture 6., and Figure 6.) The most popular object in this example is the object number 2. Each of the two models may also have a 'polygamic' variant.

Both models may easily be modified in such a way to simulate the case when each subject tries to be as different from its neighbours as possible. Mixed cases may be simulated as well.

IV. Controlled GoC with feedback between subjects and the environment

In the previous models elements of the impact matrix i_{mn} were generated randomly at the beginning of the game and then remained unchanged. In the controlled GoC models impact matrix does change - if a subject n shows at the moment $t = 0$ preference p_{mn} for an object m the environment 'puts pressure' on the subject to choose or to reject object m independently of the impact of the object itself. So, we introduce a generalization of the impact matrix so called *external impact matrix*, $e_{mn}(t)$

$$e_{mn}(t) = a * p_{mn}(t) + b * i_{mn}(t) + c * \eta(t) + d * \eta(t) \qquad (3)$$

$$p_{mn}(t+1) = p_{mn}(t) * e_{mn}(t) \qquad (4)$$

where $c_{mn}(t)$ denotes environmental pressure exerted on the subject n to choose ($c_{mn}(t) > 0$) or to reject ($c_{mn}(t) < 0$) object m; $\eta(t)$ denote noise; a, b, and d are constants (cf. Eq. (1)); again, a star denotes multiplying element by element, not multiplication of the matrices. If $c_{mn}(t)$ is not equal zero it means that the environment exerts pressure e.g. by 'taking a notice' of subject's n preferences towards the object m and passing to the subject n information about the object m that is not normally presented by the object m itself. This way subject n becomes more and more encouraged or discouraged to choose object m and the impact of the object itself (expression $b*i_{mn}(t)$ in Eq. (3)) may often be much smaller than such an environmental pressure. Coefficient a in (3) measures 'stiffness' of the subjects' own preferences.

In the following examples we consider the simplest case - all $c_{mn}(t)$ are identical and remain constant, equal c ; we also assume $b = 0$ and $d = 0$; for simplicity we also assume that each objects' subclass consists of only one object. For better comparison with natural GoC models discussed above in the following simulations we chose $a = 0.001$ and $c = 0.950$ to start with $e_{mn}(0)$ values

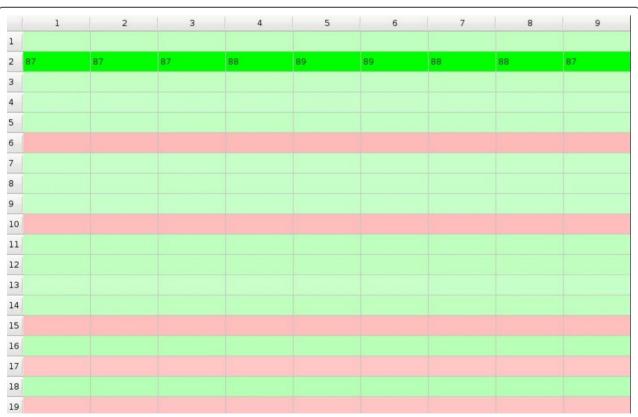**Figure 5 Natural GoC with interaction between subjects - monogamic variant with one subclass**.

Figure 6 Natural GoC with interaction between subjects - monogamic variant with the number subclass at least equal to the number of subjects.

Figure 7 Controlled GoC with feedback between subjects and the environment - monogamic variant.

from similar range as $i_{mn}(t)$ values in the previously discussed models (cf. Figure 2) while $p_{mn}(0)$ remains the same as previously (Figure 1).

We consider here two models - controlled GoC with feedback between subjects and the environment, 'monogamic' variant, without interactions between the subjects (see Additional File 5. Picture 7., and Figure 7) and with additional interactions between subject as given by Eq. (2) (see Additional File 6. Picture 8., and Figure 8). In this last case in particular final preferences may be significantly different from the initial ones. This model may work for example with Google advertisement - if one has shown for example interest about China, Google with each searching or opening of Gmail will display information like 'Big discounts for flights to China', 'Thousands cheap hotels in China' etc. If somebody has shown interests for example about Brazil, Google will display similar kind of information about Brazil, etc. This model is also the very base of political propaganda, in particular during elections. Each of the two models may also have a 'polygamic' variant.

Conclusions

In all above examples the subjects made their decisions of choosing while having only limited information about the objects they were choosing from. Moreover, each of the subjects had accessible different information, so each subject had different preferences.

Even the simplest 'monogamic model' clearly demonstrates how choosing for example a mating partner may just be a 'deterministic chaotic' game. Even miniscule differences in initial conditions (initial preferences) and/ or small differences in number of times prospective partners are exposed to the subject while looking for a partner may cause very big differences in the final outcome i.e. in the mating pairs that would be finally formed. More complicated models show that influence of the environment may cause changes of initial preferences and so formation of very different mating pairs.

In commerce influence of the social environment is even more important. E.g. while choosing a model of a car one is deciding to buy the problem which car models the neighbours possess might be of primary importance in two quite opposite ways - one may want to be 'just like others' or one may want to differ as much as possible.

But the same models also well illustrate 'games' played during any election campaign and by governments, big companies, etc in general. Famous statement attributed to Joseph Goebbels, minister of propaganda of the III
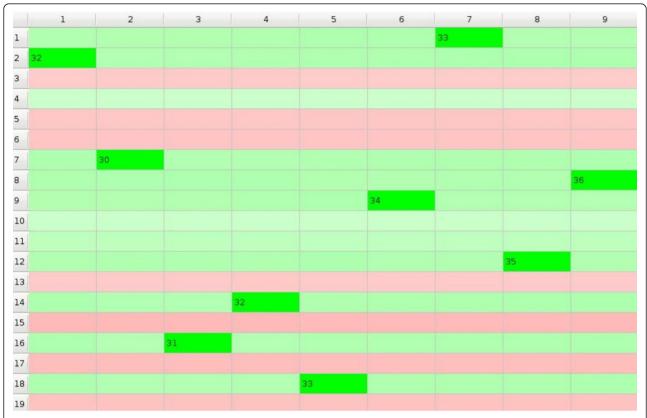
Figure 8 Controlled GoC with feedback between subjects and the environment and with interactions with other subjects - monogamic variant.

Reich, states: "If you tell a lie big enough and keep repeating it, people will eventually come to believe it...." However, most probably similar statement was even earlier formulated by Vladimir Lenin: "A lie told often enough becomes truth". These statement became a base of propaganda in all authoritarian regimes.

The father of modern Psychology William James (1842-1910) said: "There's nothing so absurd that if you repeat it often enough, people will believe it." And 'the Father of Spin' Edward L. Bernays, a nephew to Sigmund Freud, in 1920s created what is now called PR by observing how masses of people could be swayed through messages repeated over and over hundreds of times [3]. In science frequently undetected fraudulent data and results in peer-reviewed journals are quoted by other researchers, who are in turn re-quoted by still others, and so on ([4,5]).

'Regularity' that a lie if repeated hundreds times becomes the truth is similar to money savings with constant compound interests. So called 'rule 72' says that if one subdivides 72 by the percentage added per annum the result will be a good approximation of the period (expressed in years) necessary for the investment to double itself. Similarly, if one subdivides 72 by ($| i_{mn} - 1|*100$) one calculates the approximate number of expositions

necessary for the preference of the n-th subject towards the m-th object to increase twofold under assumption that the impact matrix I does not change with time. One should not forget 'negative impact' - if $i_{mn}(t) < 1$ each exposition discourages subject n from choosing object m. So, a minuscule difference in initial preferences or a minuscule difference in impact values may lead to dramatically different results of GoC even in the simplest cases.

So, *econobiophysics* does have a very broad spectrum of application.

Additional material

Authors' contributions
WK - main ideas and writing the paper. MP - main computer programming, graphics generation, model improvement. PS - programming and model improvement. RS - programming and model improvement. All authors read and approved the manuscript.

Competing interests
The authors declare that they have no competing interests.

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